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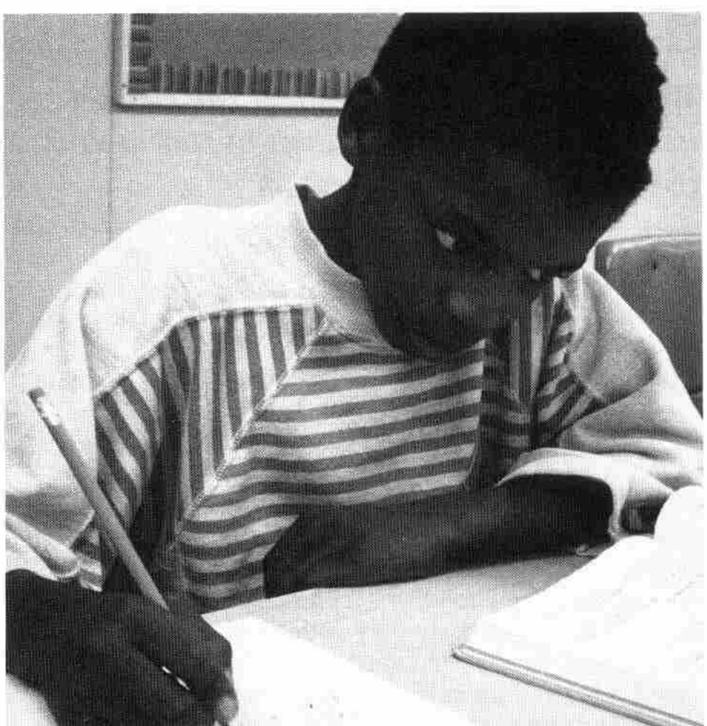
Writing, reading, and talking mathematics: One interdisciplinary possibility

The language arts and mathematics can be integrated as students write, solve, and discuss story problems. With systematic instruction in all its phases, the process keeps fifth graders motivated and successful.



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The heart of mathematics is problems. Problems guide most mathematics activity and learning. According to Kilpatrick (1985), "From the mathematical, or subject-matter, point of view, one sees mathematical problems as defining the discipline of mathematics" (p. 3). Mathematicians often grapple with abstract problems in their everyday work. Scientists, engineers, city planners, architects, and other professionals use mathematics to solve authentic problems that emerge from the challenges and tasks in their work. All of us solve mathematical problems embedded in our day to day lives, such as shopping for food, estimating distances, and playing card games. "All mathematics is created in the process of formulating and solving problems" (Kilpatrick, 1982, p. 2).

Problem writing and problem solving can provide intriguing learning situations in the mathematics classroom for both students and teachers. Our work in developing a problem-writing curriculum in mathematics draws upon two ideas from whole language theory (Edelsky, Altwerger, & Flores, 1991): First, language activity in school ought to be as meaningful and purposeful as it is outside school; second, teachers and students should collaborate in the content and direction of curriculum.

The first assumption of whole language applicable to the teaching of mathematics aims to organize school mathematics experi-

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ences in ways that resemble mathematics practice in out-of-school settings. In everyday life mathematics is a problem-finding, conjecturing, applied, purposeful, and social activity (Albers & Alexanderson, 1985; Carraher, Carraher, & Schliemann, 1985). So it makes sense that whole language classrooms would represent mathematics as problem finding, as a social activity, and as a tool to make sense of everyday life. Since problems and problematic situations are often the basis for mathematical activity outside school, problems ought to be the basis of a "whole mathematics" program. Moreover, these problems ought to reflect students' everyday experience, including content area study, classroom and school situations, and problems that reflect students' imagination.

A second assumption of whole language is the collaborative way in which students and teachers shape the curriculum. Lester (1985) concluded that teachers find problem-solving classrooms in mathematics difficult to manage because the environments tend to be less teacher directed than usual. However, we have found that elementary language arts teachers can more effectively share control over curriculum with students. For example, teachers can invite students to select their own writing topics, the mode of writing, and even audiences for their written work. When invited to draw upon personal experience in socially interactive classrooms, young writers can effectively develop their own topics (Atwell, 1987; Graves, 1983). We propose that successful mathematics curriculum collaboration between teachers and students can come in the form of teacher- and student-written mathematics problems.

In this article, we describe some of our work with students as they write, solve, and share mathematics story problems. We also provide suggestions for teachers interested in implementing a problem-writing program in mathematics.

Rationale for a problem-writing approach to elementary mathematics

Common sense suggests that students of all ages harbor a wealth of questions, problems, and confusions about themselves and their relationship with the world. Too often, however, adult interests tend to dominate curriculum and everyday life in classrooms. Reflecting Dewey (1938), we believe that one objective of education is to recognize children's natural tendency to ask questions and then to exploit this curiosity to achieve curricular ends. Duckworth (1987) affectionately refers to this as the "having of wonderful ideas," when children raise what is to them an interesting question or problem about their world and then "are moved to tax themselves to the fullest to find the answer" (p. 5). A student-centered problem-writing approach to school mathematics exploits students' natural dispositions to wonder and ask questions.

There has been periodic advocacy in the mathematics education literature for student-written story problems (Silverman, Winograd, & Strohauer, 1992; Whitin, Mills, & O'Keefe, 1990; Wirtz & Kahn, 1982) and for mathematics problem posing generally (Brown & Walter, 1983; Kilpatrick, 1987). Problem posing, a basic activity of mathematicians, reflects mathematics as a creative, generative activity (Brown & Walter, 1983; Kilpatrick, 1987; Moses, Bjork, & Goldenberg, 1990). If students are regularly invited to write their own mathematics problems, a more authentic version of mathematics for the school curriculum may be conceptualized.

Problem writing and problem solving can provide intriguing learning situations...for both students and teachers.

Finally, the National Council of Teachers of Mathematics (1991) has recommended that teachers move from traditional, text- and teacher-centered teaching to empower students to construct their own knowledge of mathematics. As teachers embrace this vision, their task will be to help students work together to make sense of mathematics; rely more on themselves to determine whether something is mathematically correct; learn to reason mathematically; learn to conjecture, invent, and solve problems; and connect mathematical ideas and applications. Socially interactive mathematics classrooms in which teachers invite students to write and solve their own mathematics story problems certainly embrace all of these practices.

Scenes from Donna Strohauer's fifth-grade classroom

What would the problem-writing instructional process look like? Here is one brief scenario. First, the teacher identifies a curricular topic in mathematics. The teacher then organizes some relevant instructional experiences, including immersion in problems that reflect this topic. If the topic can be applied to some everyday experience, the teacher helps students identify pertinent real-life situations. For example, problems that reflect decimal operations could involve money, trading cards, or shopping. Probability suggests situations like weather forecasting, betting, or games of chance.

During this exploration or prewriting phase, students may use writing, discussion, or drawing to consider how that particular mathematics topic relates to their own experience. Now with adequate background knowledge students are ready to generate coherent mathematics story problems.

Students work to solve their problems, share these problems and solutions with peers, and then revise problems and solutions based on the feedback they receive. A structure such as mathematician's circle, described below, can be used to support students' sharing. Finally, students publish their problems on worksheets, in anthologies, or in some other formal manner for an audience of classmates, students from other classes, or some other group.

Donna Strohauer, a fifth-grade teacher in Greeley, Colorado, has been experimenting with aspects of this problem-writing approach to mathematics for several years. Her mathematics class usually begins with "mathematician's circle" (a variation of Graves and Hansen's [1983] author's chair): One student shares a self-generated story problem with the whole class, small groups, or individual peers. The problem writer directs peers to "take a few minutes and see if you can do my problem."

What usually happens at this point includes the following: (a) students call out answers, (b) students ask for explanations of answers, (c) either the problem writer or volunteers go to the chalkboard to lead explanations, (d) students offer alternative explanations, or (e) Donna intervenes to address some academic or behavioral issue.

After understandings are reached regarding students' solutions, the problem writer asks peers "What did you like about my problem?" and "How can I change the problem?" Students respond for a few minutes. Mathematician's circle is often a freewheeling affair of negotiated solutions and new questions as well as criticism of shared problems (usually framed in language such as "What I like about your problem was...").

Figure 1
Barb's bubble gum problem

I boght a pack of bubble gum for $1.50. In the pack of gum there's 10 peices of gum. Each piece has 5 bubbles. How much does each bubble cost?

Several vignettes illustrate mathematics instruction for Donna's students.

Vignette one. Barb's bubble gum problem (see Figure 1) had just been solved in a whole-class mathematician's circle.

Next, the students explored possibilities for new problems.

Barb: How can I change this problem? Celene?
Celene: Um...you could put a lot more bubbles in it?
Barb: Yeah. Krista?
Krista: You buy more than one pack.
Barb: Yeah. Danny?
Danny: You could raise the money on it. You could put more, put extra information. You don't have any extra information in it.
Barb: I know. Jenny?
Jenny: You could put I buy...um five pieces of gum each day and the taste lasts for 12 hours each day, or something like that.
Barb: Nikki?
Nikki: Um...you could put like...how much does each pack of bubbles cost?
Barb: Anyone else? Travis?
Travis: Well, you could say, I went on a trip and each...gum is good for 10 bubbles...it's so hot where you are so it only blows 3 bubbles.
Barb: Yeah, that's a neat problem...or like, like you can save, save the gum and each time you put it back in your mouth, you might [inaudible] 3 bubbles. [Bruce says, "Oh."] Bruce?
Bruce: Every bubble takes like 80 grams of air, or something like that. Like, how many grams will I need in one pack?

Generating and solving new problems from old, previously solved problems helps learners deepen mathematical understanding and integrate new and old knowledge. Furthermore, old problems are a fertile source of new problems (Brown & Walter, 1983; Polya, 1957). As they changed or added new problem conditions, these students exhibited a certain insouciance, almost like they were involved in an intellectual play on quantities and words that represented those quantities. The interest and fluency with which students raised new ideas for Barb's problem was typical of this class during mathematican's circle.

Vignette two. Bruce was ready to write a problem. He quickly generated a topic that reflected one of his hobbies, long distance

Figure 2
Bruce's biking problem

If I biked across Kanses and colorado I would go about 600 the next summer I would try to go about 700 then I would try to up it to 900 IF I did that amout of rideing every 3 summers how long would it take to get a totel of what it would be to go both way across the unided states.
Look at the map of the unided states to see how far it is.

biking (see Figure 2). The school principal had biked across the United States the previous summer, and the school community was buzzing with this accomplishment.

Bruce reflected aloud: "I'm trying to think of the problem. I'd like to get 'em going with exercise. Or something like that. So...I could go with a...bicycle trip and then a bicycle trip across Kansas maybe...if I rode a bicycle into Kansas and Colorado it'd be about 500 miles or so and I could make it so I could see if I did better per year for 3 years maybe I could get 'em...."

Once Bruce identified his general topic and a tentative culminating question, he was ready to begin writing. He wrote: *If I biked across Kanses and colorado I would go about 600....* He was unsure what to write next. He said, "miles...if I went 600 miles...I'm trying to think how I can...what I can use that for...every summer or something...600 miles maybe, and then I can expand it to 700 the next summer and then so on...I can do that...." He wrote, *the next summer I would try to go about 700 (miles) then I would try to....*

Suddenly he stopped writing and said, "If I did that for four summers, how long would it take me to...wait! Every three summers, if I did that, how long would it take me to go both ways across...how long would it take me to get um a total of going both ways across the United States." Bruce was now ready to complete writing the problem: *up it to 900 If I did that amount of rideing every 3 summers how long would it take to get a totel of what it would be to go both way across the unided states.*

Finally, he reread the problem aloud. "I don't know if it would make sense to them, though...I could give a hint to look at a map of the United States to see how far it is?" He wrote: *Look at the map of the unided states to see how far it is*. Bruce then solved the problem with no observable hesitation.

In this episode, the writing was more difficult than the solution. Bruce waited to generate a final, culminating question, particularly a question that would be challenging for his peers or at least interesting to himself. His first idea for a question was ambiguous. "I could make it so I could see if I did better per year for 3 years maybe I could get 'em." After he increased the mileage each of the next two summers, a question about distance across the United States made sense, particularly in light of the principal's cross-country trip. The originality and coherence of his question was impressive. Bruce's awareness of audience, a characteristic of mature writers (Flower & Hayes, 1980), is evident by his comment about "getting 'em going with exercise" and, later, the hint about using the map to figure cross-country distance. His developing sense of audience was stimulated by the everyday regularity with which students shared problems.

The expectation that they were going to be writing mathematics story problems almost every day also influenced students to think about story problems outside school. One student, whose father sanded local streets during snowstorms, noticed his father's activity during one storm, interviewed the father about this experience, and

Figure 3
Robbie's year problem

It is the year 1836. How many days until 1989?

then wrote a problem about it. One parent reported that her son kept a chalkboard in the basement on which he wrote problems nightly; he would go downstairs late at night for this purpose whenever he had insomnia. Another student reported that she generated a story problem during a boring church sermon. Donna also noticed students infusing an unusual amount of informal mathematical conversation in places such as the lunch line or on the bus, conversation marked by the comment, "Hey, that would make a good story problem."

Vignette three: Robbie shared his year problem with Jeff and Steve (see Figure 3). Although Jeff had solved this problem earlier, this was Steve's first encounter with it.

Robbie:	[reads problem] It is the year 1836. How many more days until 1989?
Steve:	Ok, but you can't do this 'cause you don't know what day it is.
Jeff:	So! You have the year already. You just start a fresh year.
Steve:	How many days, he said.
Jeff:	I know—
Robbie:	You start from the first year...February-January is the first....
Jeff:	That's what we mean.
Steve:	Oh, oh.
Jeff:	Ok, you want me to tell you the first step?
Steve:	No. Yeah.
Robbie:	How many days are in a year?
Steve:	No!
Jeff:	Let's see if he can get it. [Steve subtracts the two years; gets 153.]
Jeff:	Now what do you do?
Steve:	(laughs) 153 times 365....
Robbie:	It's not easy.

Initially, Steve seemed to challenge the coherence of the problem, perhaps the assumption that the question asked for the number of days from the first day of 1836 to the first day of 1989. Robbie explained that the solver is to start with the first day in January. Robbie's question, "How many days are in a year," may have been the significant cue that helped Steve generate an understanding of the solution. Steve's quick rejection of help suggests that, in his mind, he made the satisfactory connection immediately after Robbie's prompt. In this episode, the help of those "in the know" came in the form of comments and questions indirectly related to the solution.

Vignette four. John, a third grader in another problem-writing classroom, wrote a problem that was rich in possibilities for reading, writing, and mathematics learning and teaching (see Figure 4).

Figure 4
John's shopping problem

John April 25
Ciris road his Newscate board to the International Bike and gift to get some Knee pads. When he got there he saw that they wern't on sale any more. And he had onily Brogh 8:00. And the knee pads were 5:00 plus tax which was 5:95. then he saw a sun Rad helmit. for 8:00. he wanted Both and he oniy had 8:00. So the stor manger said "if you clean up my store I will give you 11:00!! But 11:00 Dallers would notto cover it at all. how much more money would He need to get them Both.

When John shared this problem, almost half of the students interpreted the tax to be \$5.95. The other students, apparently possessing a more developed understanding of the concept of tax, understood it to be 95 cents, since the tax could not be more than the purchase price itself. A heated argument between the two groups ensued, and John was called upon to clarify his meaning.

Afterwards, the teacher used this problem as the basis for lessons on how to receive feedback from others in order to facilitate evaluation, revision strategies, and the role of background knowledge in reading. Other lessons related to John's problem could focus on certain spelling patterns, use of arrows to add information, writing money notations, spacing between words, solving multistep problems, and adding and subtracting money.

Discussion

When invited and trusted to do so, these students were willing to initiate and sustain their own mathematics problem writing, reading, and solving with enthusiasm and thoughtfulness. We propose two possible explanations for their behavior. First, the writing genre, story problem, is relatively simple to produce, even for the most inexperienced writer. The writer needs only a general topic, one or two problem conditions, and a culminating question. Second, student authors had interested audiences who listened to, solved, and questioned their problems. Students' participation in a highly interactive community—of problem finders and problem solvers, of readers and writers, of listeners and talkers, and of critics and collaborators—may have motivated them to write interesting and challenging problems.

Engaging students as problem writers in mathematics suggests a conceptualization of writing as a tool for learning (Emig, 1977), in this case a tool to deliberately reflect upon mathematical experiences and imaginings. By embedding mathematics learning in story, a form of discourse familiar and enjoyable to students, their inclination to return to their mathematical stories and problems is greater than when the mathematics is disembodied as it is in many textbooks. When students return to their own stories, especially as they are called upon to explain the meanings of problems to peers, the conditions for mathematical learning increase. Written mathematics stories and story problems have the potential to freeze students' existing conceptions of mathematics concepts and experiences, thus permitting them to deliberately and collaboratively inspect their constructions and, with others, revise understandings and even misconceptions.

Elementary students often approach school mathematics problems impulsively, attending to surface features of the problem situation in order to decide what action to take. Students see traditional problems "as school tasks rather than as an intellectual challenge that is worth accepting" and often "grab at answers so as to escape from the task as fast as possible" (Kilpatrick, 1985, p. 12). When students write and solve their own problems, it becomes more difficult to rely on just the surface features of the problems. This was certainly evident in vignette three when Steve attempted to solve Robbie's problem as well as in vignette four when the argument surfaced regarding the sales tax. Discussions among problem writers and solvers led to a rich conceptualization of the problems and their solutions.

Suggestions for implementation

Certain instructional conditions can encourage students to write problems that are moderately challenging, contain viable mathematics, and are embedded in everyday experience or imagination. Six conditions, based on Cambourne's (1988) work, can support students' development as problem writers in mathematics. (See Winograd [1993] for an elaboration of these conditions.)

First, students should be immersed in the types of problems that we expect them to write. Examples of these problems as well as topics for problems should come from the teacher, students, and the formal curriculum. All the content strands for mathematics articulated in the *Curriculum and Evaluation Standards* (National Council of Teachers of Mathematics, 1989) can be effectively addressed in a problem-solving, problem-posing curriculum.

Second, students benefit from their teacher's demonstrations on how to find topics for problems and then how to write those problems. Teachers need to share how they use mathematics to make sense of their lives and how mathematics is a tool that can be used to solve both real and imagined problems.

Third, students need control over the mathematical and nonmathematical content of their problems.

Fourth, students must practice mathematics problem writing and solving every day. The posing and solution of mathematics problems ought to permeate students' school mathematics experience as much as it permeates their everyday lives. Students need enough time during the day to develop the concepts and dispositions of problem writing and solving.

Fifth, students profit from regular feedback from peers and the teacher as to the meanings of their problems and solutions. Peers provide writers with an interested or critical audience for their work. Furthermore, by regularly teaching or defending self-generated problems to peers and also by solving problems written by peers, students' problems assume equal status with those written by adults.

Finally, problem writing as a core component of the school mathematics program can lead to students' learning only when it is coupled with a well articulated curriculum for the teaching of mathematical concepts and problem-solving strategies. Students will become discouraged if they do not know how (or do not have the confidence) to solve or understand their problems or they do not have the skills to explain their understandings to others. Adequate background knowledge in mathematics will enable students to generate coherent story problems.

Conclusion

Allowing students to write and solve their own problems promotes more effective problem-solving performance with elementary (Keil, 1965), secondary (Bell & Bell, 1985), and adult learners of mathematics (Gage, 1984). However, we maintain that students' wholehearted engagement as problem writers and problem solvers does not by itself lead to their learning as writers, readers, or mathematicians. Although engaging students as the authors of mathematics story problems stimulates a more friendly context for learning, systematic and regular teaching of problem-writing and problem-solving knowledge must accompany the invitation to write.

Perhaps one of the major benefits of engaging students in problem writing and solving is what teachers can learn about their students. Not only do teachers see the mathematics their students feel most comfortable using, but they also gain insights into how their students are able to convey their understandings and thoughts in writing. "It is through what children actually conceive, organize, do, and present to us and others that their intellectual functioning can be seen and understood most clearly" (Stevenson & Carr, 1993, p. 21).

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